STRESS ANALYSIS OF THICK WALLED CYLINDER

A thesis

Submitted by

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in
MECHANICAL ENGINEERING

Under the guidance of

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CERTIFICATE

This is to certify that this report entitled, "**Stress analysis of thick walled cylinder**" submitted by **Susanta Choudhury (109ME0365)** in partial fulfillment of the requirement for the award of Bachelor of Technology Degree in Mechanical Engineering at National Institute of Technology, Rourkela is an authentic work carried out by them under my supervision.

To the best of my knowledge, the matter embodied in this report has not been submitted to any other university/institute for the award of any degree or diploma

Date:

Dr. H Roy
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ACKNOWLWDGEMENT

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Grateful acknowledgement is made to all the staff and faculty members of Mechanical Engineering Department, National Institute of Technology, Rourkela for their encouragement. In spite of numerous citations above, the author accepts full responsibility for the content that follows.

Susanta Choudhury

ABSTRACT

It is proposed to conduct stress analysis of thick walled cylinder and composite tubes (Shrink fits) subjected to internal and external pressure. Many problems of practical importance are concerned with solids of revolution which are deformed symmetrically with respect to the axis of revolution. The examples of such solids are: circular cylinders subjected to uniform external and internal pressure. The stress analysis of thick walled cylinders with variable internal and external pressure is predicted from lame's formulae.Different case in lame's formula arethick walled cylinder having both (a) External and Internal pressure (b) Only Internal Pressure (c) Only External Pressure. In case of Composite tubes (Shrink Fit) the contact pressure between the two cylinders is determined then stress analysis is done by applying external and internal pressure in tube by lame's formulae.Theoretical formulae based results are obtained from MATLAB programs. The results are represented in form of graphs.

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NOTATIONS

σ_z	Plane stress in z-axis
σ_r	Radial Stress
σ_θ	Hoop Stress
τ_{rx}	Shear Stress in rx-plane
τ_{ry}	Shear Stress in ry-plane
τ_{rz}	Shear Stress in rz-plane
ε_z	Strain in z-direction
ε_θ	Circumferential strain
ε_r	Radial strain
E	Young’s modulus
ν	Poission ratio
P_a	Internal Pressure
P_b	External Pressure
P_c	Contact Pressure

CHAPTER-1
INTRODUCTION

1.1 Problem statement:

Thick walled cylinders are widely used in chemical, petroleum, military industries as well as in nuclear power plants. They are usually subjected to high pressure & temperatures which may be constant or cycling. Industrial problems often witness ductile fracture of materials due to some discontinuity in geometry or material characteristics. The conventional elastic analysis of thick walled cylinders to final radial & hoop stresses is applicable for the internal pressure up to yield strength of material.

General applicationn of Thick- Walled cylinders include, high pressure reactor vessels used in mettalurgical operations, process plants, air compressor units, pneumatic reservoirs, hydraulic tanks, storage for gases like butane LPG etc.

In this Project we are going to analyze effect of internal and External Pressure on Thick walled cylinder , How radial stress & hoop Stress will vary with change of radius. Contact pressure in shrink Fit and it’s affect on hoop stress and radial stress in analysed.

LITERATURE REVIEW

Xu& Yu [1] carried down shakedown analysis of internally pressurized thick walled cylinders, with material strength differences. Through elasto-plastic analysis, the solutions for loading stresses, residual stresses, elastic limit, plastic limit & shakedown limit of cylinder are derived.

Hojjati&Hossaini [2] studied the optimum auto frittage pressure & optimum radius of the elastic plastic boundary of strain hardening cylinders in plane strain and plane stress conditions. They used both theoretical and & Finite element modelling. Equivalent von-Mises stress is used as yield criterion.

M. Imanijed& G. Subhash[3] developed a generalized solution for small plastic deformation of thick- walled cylinders subjected to internal pressure and proportional loading.

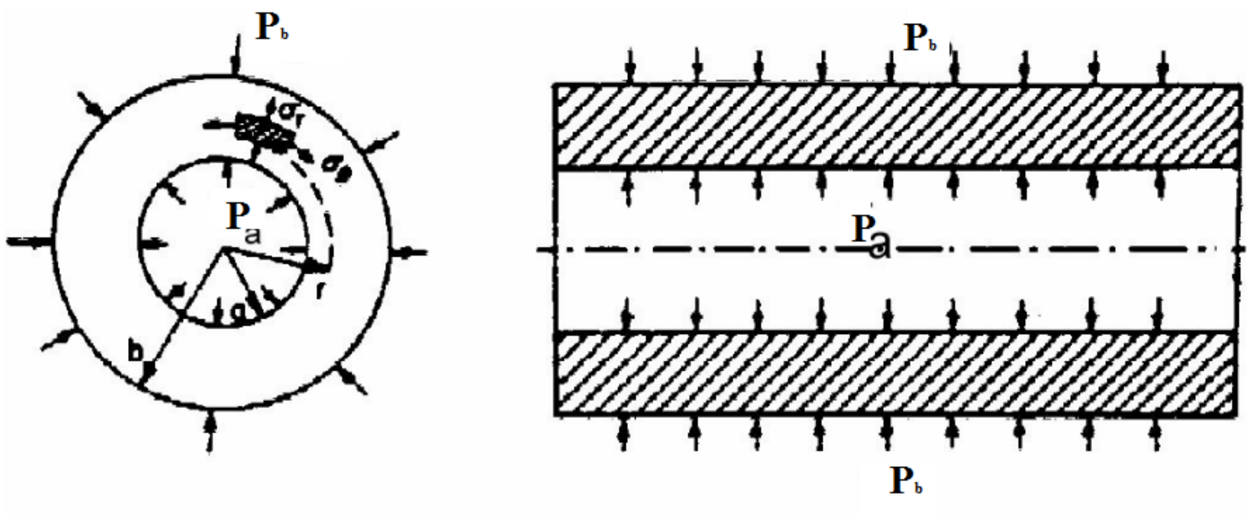
Y.Z. Chen & X.Y. Lin [4] gave an alternative numerical solution of thick walled cylinder and spheres made of functionally graded materials.

Li &Anbertin [5] presented analytical solution for evaluation of stresses around a cylinder excavation in an elastoplastic medium defined by closed yield surface.

CHAPTER 2
MATHEMATICAL MODELLING

2.1 LAME'S PROBLEM-Thick walled cylinder subjected to internal and external pressure

Consider a cylinder of inner radius a and outer radius b. Let the cylinder to be subjected to internal pressure $\boldsymbol{P_a}$and external pressure $\boldsymbol{P_b}$. It will have two cases plane stress case ($\boldsymbol{\sigma_z}$**=0**)or as a plain strain case ($\boldsymbol{\varepsilon_z} = \mathbf{0}$)



2.1. a. Plane Stress

Let the ends of the cylinder be free to expand. We shall assume that σ_z=0 ours results just justify this assumption . Owing to uniform radial deformation τ_{rz} =0, Neglecting body forces we can write

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_{r-}\sigma_\theta}{r} = 0$$

Since r is the only independent variable the aboveequation can be written as

$$\frac{d}{dr}(r\sigma_r) - \sigma_\theta = 0 \qquad \text{----------------eq(1)}$$

From Hooke's Law:

$$\varepsilon_r = \frac{1}{E}(\sigma_r - \nu\sigma_\theta)$$
$$\varepsilon_\theta = \frac{1}{E}(\sigma_\theta - \nu\sigma_r)$$

Stresses in terms of strain

$$\sigma_r = \frac{E}{1-\nu^2}(\varepsilon_r + \nu\varepsilon_\theta)$$
$$\sigma_\theta = \frac{E}{1-\nu^2}(\varepsilon_\theta + \nu\varepsilon_r)$$

After putting values ε_r and ε_θ

$$\sigma_r = \frac{E}{1-\nu^2}\left(\frac{du_r}{dr} + \nu\frac{u_r}{r}\right)$$
$$\sigma_\theta = \frac{E}{1-\nu^2}\left(\frac{u_r}{r} + \nu\frac{du_r}{dr}\right) \quad \text{--eq(2)}$$

Substituting above values in equation (1), we will get

$$\frac{d}{dr}\left(r\frac{du_r}{dr} + \nu u_r\right) - \left(\frac{u_r}{r} + \nu\frac{du_r}{dr}\right) = 0$$

$$\frac{du_r}{dr} + r\frac{d^2u_r}{dr^2} + \nu\frac{du_r}{dr} - \frac{u_r}{r} - \nu\frac{du_r}{dr} = 0$$

$$\frac{d^2u_r}{dr^2} + \frac{1}{r}\frac{du_r}{dr} - \frac{u_r}{r^2} = 0$$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(u_r r)\right] = 0$$

u_r can be found from this equation as

$$u_r = C_1 r + \frac{C_2}{r}$$

Substituting this values in Eq. (2)

$$\sigma_r = \frac{E}{1-\nu^2}\left[C_1(1+\nu) - C_2(1-\nu)\frac{1}{r^2}\right]$$
$$\sigma_r = \frac{E}{1-\nu^2}\left[C_1(1+\nu) + C_2(1-\nu)\frac{1}{r^2}\right]$$

C_1 and C_2 are constants of integration and can be found out by applying boundary conditions.

When r=a,

σ_r=-p_a

When r=b,

σ_r=-p_b

so

$$\frac{E}{1-\nu^2}\left[C_1(1+\nu) - C_2(1-\nu)\frac{1}{r^2}\right] = -p_a$$
$$\frac{E}{1-\nu^2}\left[C_1(1+\nu) + C_2(1-\nu)\frac{1}{r^2}\right] = -p_b$$

On Solving,

$$C_1 = \frac{1-\nu}{E}\frac{p_a a^2 - p_b b^2}{b^2 - a^2}$$

$$C_2 = \frac{1+\nu}{E}\frac{a^2 - b^2}{b^2 - a^2}\left(p_a - p_b\right)$$

On substituting these values we get,

$$\sigma_r = \frac{p_a a^2 - p_b b^2}{b^2 - a^2} - \frac{a^2 b^2}{r^2}\frac{p_a - p_b}{b^2 - a^2}$$

$$\sigma_\theta = \frac{p_a a^2 - p_b b^2}{b^2 - a^2} + \frac{a^2 b^2}{r^2}\frac{p_a - p_b}{b^2 - a^2}$$

2.1. a. i. Cylinder Subjected to Internal Pressure only

In this case $p_b=0$ and $p_a=p$.

Hence,

$$\sigma_r = \frac{p\ a^2}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right)$$

$$\sigma_\theta = \frac{p\ a^2}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

These equations show that σ_r is always a compressive stress and σ_θ is a tensile stress.

2.1. a. ii. Cylinder subjected to external pressure only

In this case $p_a= 0$ and $p_b = p$

Hence,

$$\sigma_r = -\frac{p\ b^2}{b^2 - a^2}\left(1 - \frac{a^2}{r^2}\right)$$

$$\sigma_\theta = \frac{p\ b^2}{b^2 - a^2}\left(1 + \frac{a^2}{r^2}\right)$$

2.1. b. Plain Strain

For long cylinder stresses are calculated as sate of plane strain.

$\boldsymbol{\sigma_z}$ presumed does not vary along the z axis.

$$\frac{d}{dr}(r\sigma_r) - \sigma_\theta = 0 \qquad \text{------------------eq(3)}$$

From Hooke's law

$$\varepsilon_r = \frac{1}{E}\left[\sigma_r - \nu(\sigma_\theta + \sigma_z)\right]$$

$$\varepsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \nu(\sigma_r + \sigma_z)\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_r - \nu(\sigma_r + \sigma_\theta)\right]$$

As $\boldsymbol{\varepsilon_z}$**=0**

$$\sigma_z = \nu(\sigma_r + \sigma_\theta)$$
$$\varepsilon_r = \frac{1+\nu}{E}\left[(1-\nu)\sigma_r - \nu\sigma_\theta\right]$$
$$\varepsilon_\theta = \frac{1+\nu}{E}\left[(1-\nu)\sigma_\theta - \nu\sigma_r\right]$$

While solving $\boldsymbol{\sigma_r}$ and $\boldsymbol{\sigma_\theta}$

$$\sigma_\theta = \frac{E}{(1-2\nu)(1+\nu)}\left[\nu\varepsilon_r + (1-\nu)\varepsilon_\theta\right]$$
$$\sigma_r = \frac{E}{(1-2\nu)(1+\nu)}\left[(1-\nu)\varepsilon_r + \nu\varepsilon_\theta\right]$$

Putting values of $\boldsymbol{\varepsilon_r}and\boldsymbol{\varepsilon_\theta}$

$$\sigma_\theta = \frac{E}{(1-2\nu)(1+\nu)}\left[\nu\frac{du_r}{dr} + (1-\nu)\frac{u_r}{r}\right]$$
$$\sigma_r = \frac{E}{(1-2\nu)(1+\nu)}\left[(1-\nu)\frac{du_r}{dr} + \nu\frac{u_r}{r}\right]$$ -----------------------------eq(4)

Substituiting these in the equation of equilibrium

$$\frac{d}{dr}\left[(1-\nu)r\frac{du_r}{dr} + \nu u_r\right] - \nu\frac{du_r}{dr} - (1-\nu)\frac{u_r}{r} = 0$$

$$\frac{du_r}{dr} + r\frac{d^2u_r}{dr^2} - \frac{u_r}{r} = 0$$

$$\frac{d}{dr}\left(\frac{du}{dr} + \frac{u_r}{r}\right) = 0$$

We can write

$$u_r = C_1 r + \frac{C_2}{r}$$

Putting these values in equation(4)

$$\sigma_\theta = \frac{E}{\left(1-2\nu\right)\left(1+\nu\right)}\left[C_1 + (1-2\nu)\frac{C_2}{r^2}\right]$$

$$\sigma_r = \frac{E}{\left(1-2\nu\right)\left(1+\nu\right)}\left[C_1 - (1-2\nu)\frac{C_2}{r^2}\right]$$

Boundary conditions

When r=a,

σ_r=-p_a

When r=b,

σ_r= -p_b,

We can get

$$\frac{E}{\left(1-2\nu\right)(1+\nu)}\left[C_1 - (1-2\nu)\frac{C_2}{a^2}\right] = -p_a$$

$$\frac{E}{(1-2\nu)(1+\nu)}\left[C_1 + (1-2\nu)\frac{C_2}{b^2}\right] = -p_b$$

and solving we can find out

$$C_1 = \frac{\left(1-2\nu\right)\left(1+\nu\right)}{E}\frac{p_b b^2 - p_a a^2}{a^2 - b^2}$$

$$C_2 = \frac{1+\nu}{E}\frac{\left(p_b - p_a\right)a^2 b^2}{a^2 - b^2}$$

Substituiting thse values we can find out that

$$\sigma_r = \frac{p_a a^2 - p_b b^2}{b^2 - a^2} - \frac{a^2 b^2}{r^2}\frac{p_a - p_b}{b^2 - a^2}$$

$$\sigma_\theta = \frac{p_a a^2 - p_b b^2}{b^2 - a^2} + \frac{a^2 b^2}{r^2}\frac{p_a - p_b}{b^2 - a^2}$$

RESULTS AND DISCUSSIONS

Program for plotting graph between radial stress and radius for thick walled cylinder subjected to internal and external pressure

```
close all
clear all


pa=17000*10^3;
pb=1000*10^3;
a=0.04;
b=0.08;
r=[a:(b-a)/1000:b];
n=1;
while(n<=1001)
sigma_r(n)= ((pa*a^2 - pb*b^2)/(b^2 - a^2)) - (((a^2*b^2)/r(n)^2)*((pa-
pb)/(b^2-a^2)));
sigma_t(n)= ((pa*a^2 - pb*b^2)/(b^2 - a^2)) + (((a^2*b^2)/r(n)^2)*((pa-
pb)/(b^2-a^2)));
n=n+1;
end
plot(r,sigma_t)
xlabel('r')
ylabel('sigma_t')
```

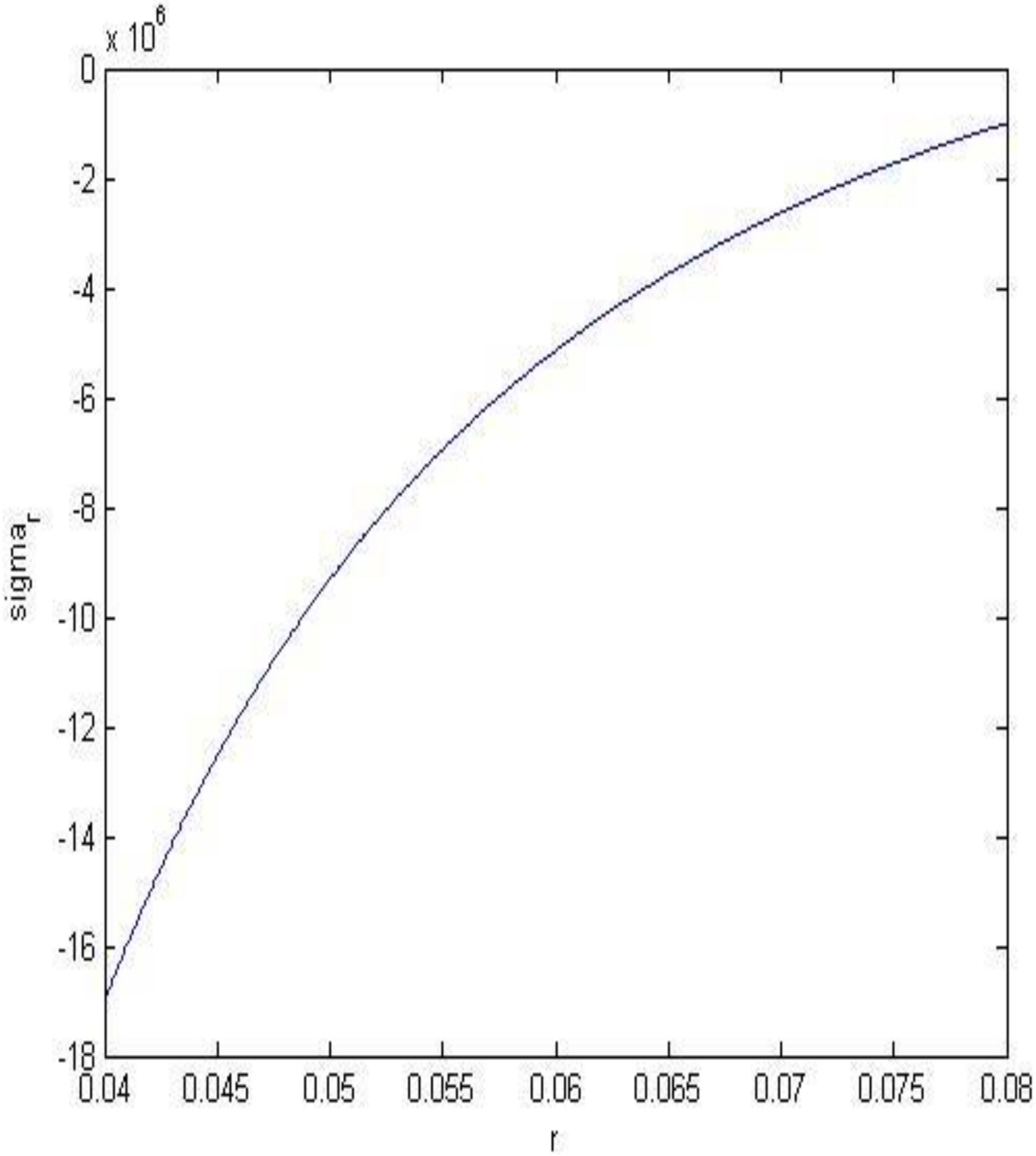


Fig. 3.1: Variation of radial stress along the radius subjected to internal and external pressure

The graph above shows the variation of radial stress along the radius of thick walled cylinder subjected to internal and external pressure. The graph shows that radial stress in this case is a compressive stress as its magnitude is negative throughout the graph.

Program for plotting graph between hoop stress and radius for thick walled cylinder subjected to internal and external pressure

```
close all
clear all


pa=17000*10^3;
pb=1000*10^3;
a=0.04;
b=0.08;
r=[a:(b-a)/1000:b];
n=1;
while(n<=1001)
sigma_r(n)= ((pa*a^2 - pb*b^2)/(b^2 - a^2)) - (((a^2*b^2)/r(n)^2)*((pa-pb)/(b^2-a^2)));
sigma_t(n)= ((pa*a^2 - pb*b^2)/(b^2 - a^2)) + (((a^2*b^2)/r(n)^2)*((pa-pb)/(b^2-a^2)));
n=n+1;
end
plot(r,sigma_t)
xlabel('r')
ylabel('sigma_t')
```

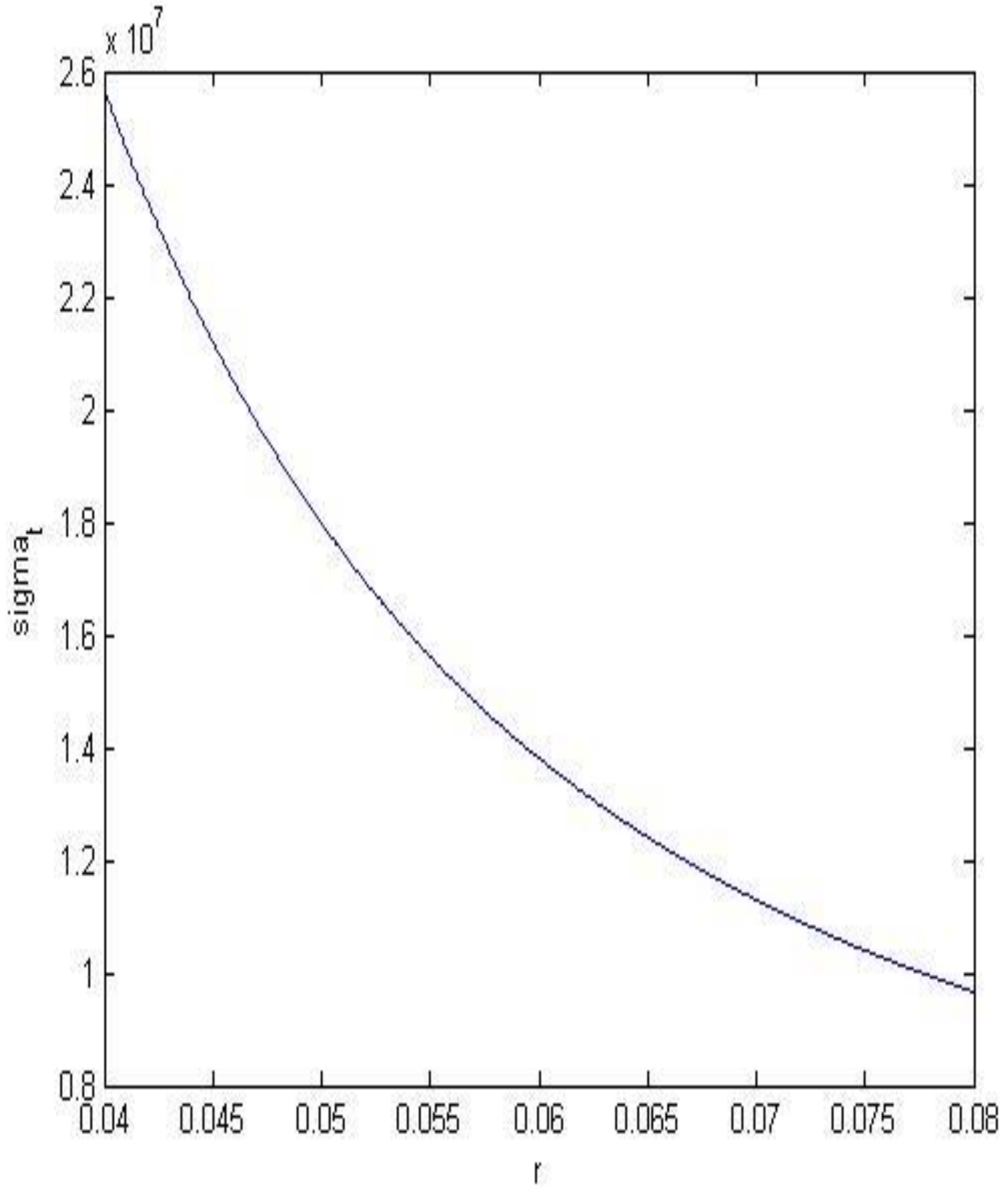


Fig. 3.2: variation of hoop stress along radius subjected to internal and external pressure

The graph above shows the variation of hoop stress along the radius of thick walled cylinder subjected to internal and external pressure. The graph shows that hoop stress in this case is a tensile stress as its magnitude is positive throughout the graph.

Program for plotting graph between radial stress and radius for thick walled cylinder subjected to internal pressure only

```
close all
clear all
pa=17000*10^3;
pb=0;
a=0.04;
b=0.08;
r=[a:(b-a)/1000:b];
n=1;
while(n<=1001)
sigma_r(n)= ((pa*a^2 - pb*b^2)/(b^2 - a^2)) - (((a^2*b^2)/r(n)^2)*((pa-pb)/(b^2-a^2)));
sigma_t(n)= ((pa*a^2 - pb*b^2)/(b^2 - a^2)) + (((a^2*b^2)/r(n)^2)*((pa-pb)/(b^2-a^2)));
n=n+1;
end
plot(r,sigma_r)
xlabel('r')
ylabel('sigma_r')
```

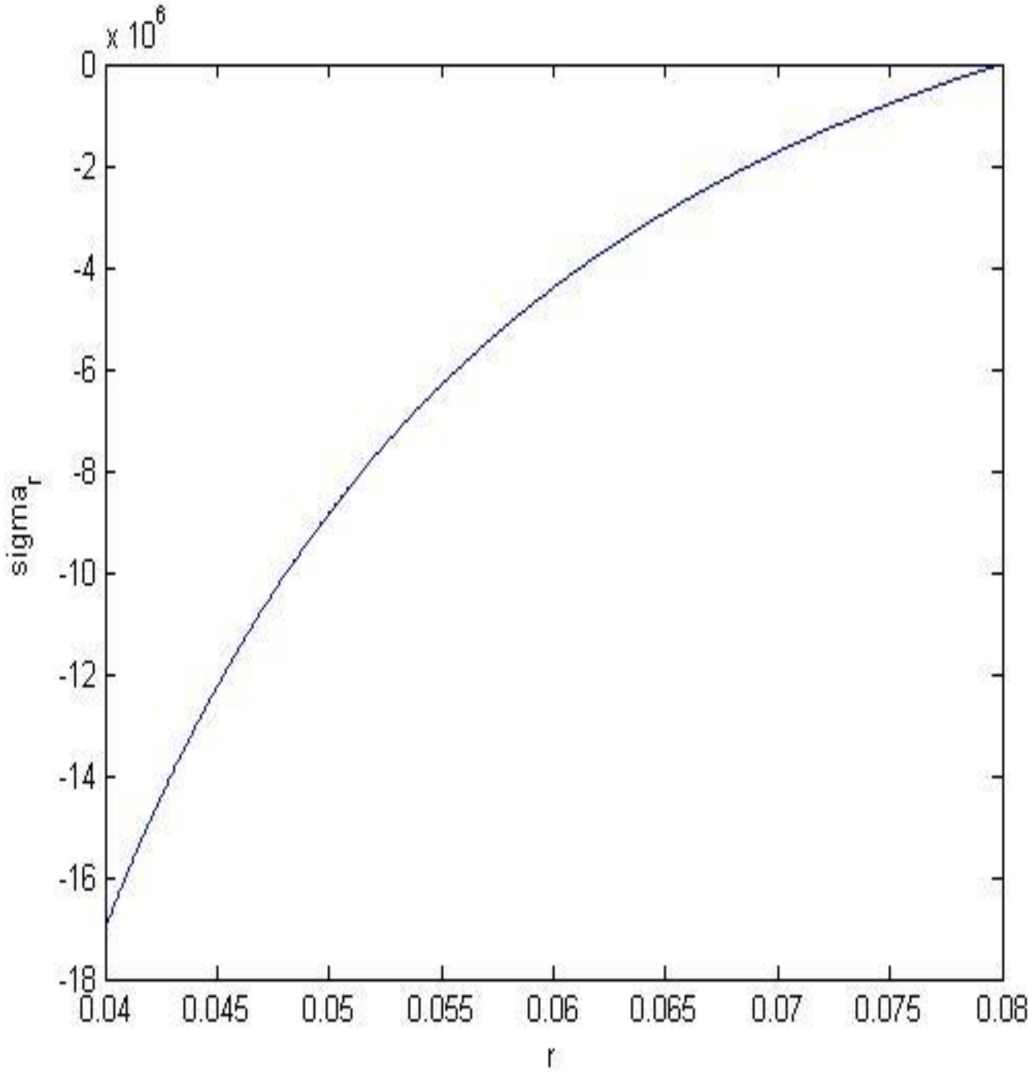


Fig 3.3: Variation of radial stress along radius subjected to internal pressure only.

The graph above shows the variation of radial stress along the radius of thick walled cylinder subjected to internal pressure. The graph shows that radial stress in this case is a compressive stress as its magnitude is negative throughout the graph.

Program for plotting graph between hoop stress and radius for thick walled cylinder subjected to internal pressure only

```
close all
clear all

pa=17000*10^3;
pb=0;
a=0.04;
b=0.08;
r=[a:(b-a)/1000:b];
n=1;
while(n<=1001)
sigma_r(n)= ((pa*a^2 - pb*b^2)/(b^2 - a^2)) - (((a^2*b^2)/r(n)^2)*((pa-pb)/(b^2-a^2)));
sigma_t(n)= ((pa*a^2 - pb*b^2)/(b^2 - a^2)) + (((a^2*b^2)/r(n)^2)*((pa-pb)/(b^2-a^2)));
n=n+1;
end
plot(r,sigma_r)
xlabel('r')
ylabel('sigma_r')
```

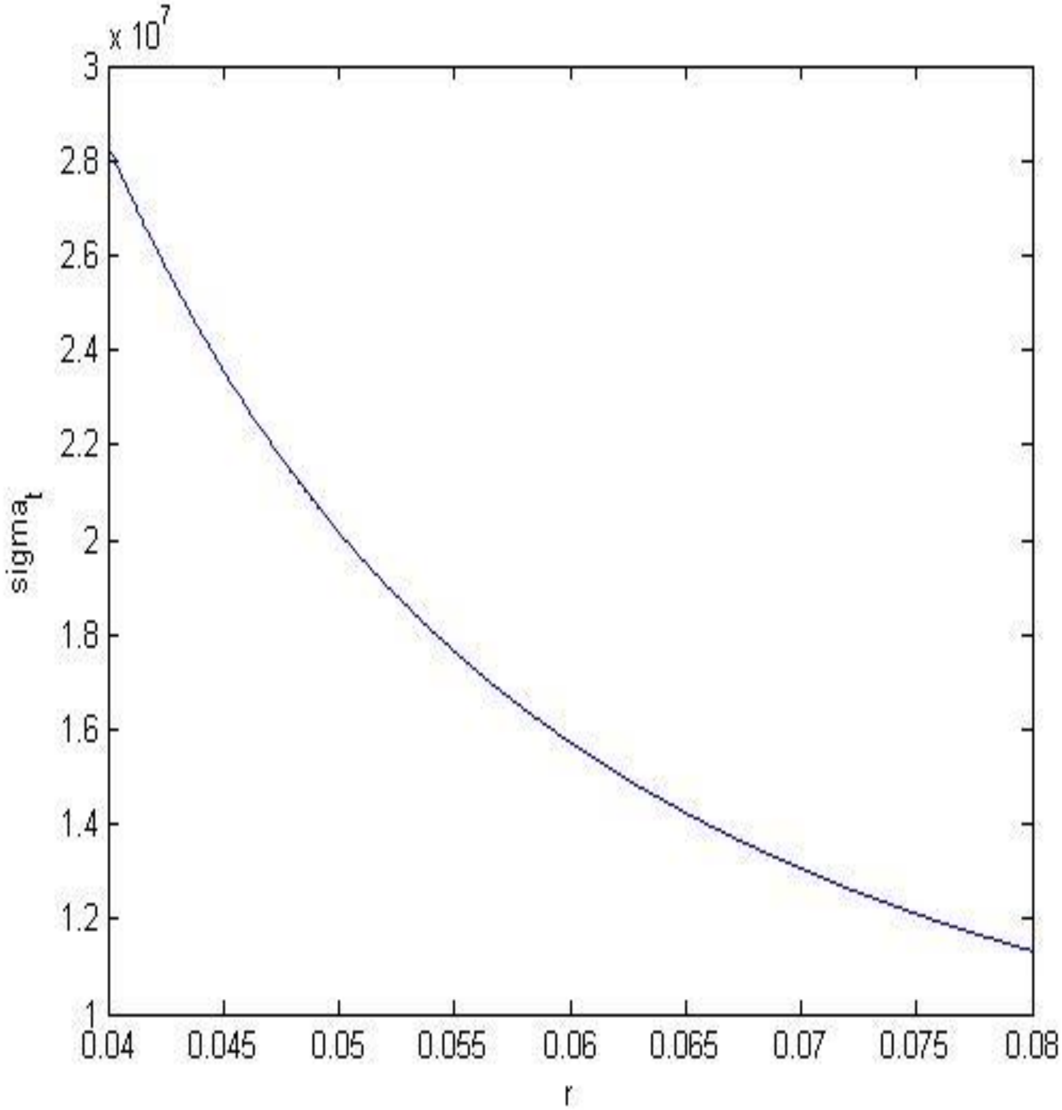


Fig 3.4:Variation of hoop stress along radius subjected to internal pressure only.

The graph above shows the variation of hoop stress along the radius of thick walled cylinder subjected to internal pressure. The graph shows that hoop stress in this case is a tensile stress as its magnitude is positive throughout the graph.

Program for plotting graph between radial stress and radius for thick walled cylinder subjected to external pressure only

```
close all
clear all

pa=0;
pb=1000*10^3;
a=0.04;
b=0.08;
r=[a:(b-a)/1000:b];
n=1;
while(n<=1001)
sigma_r(n)= ((pa*a^2 - pb*b^2)/(b^2 - a^2)) - (((a^2*b^2)/r(n)^2)*((pa-pb)/(b^2-a^2)));
sigma_t(n)= ((pa*a^2 - pb*b^2)/(b^2 - a^2)) + (((a^2*b^2)/r(n)^2)*((pa-pb)/(b^2-a^2)));
n=n+1;
end
plot(r,sigma_t)
xlabel('r')
ylabel('sigma_t')
```

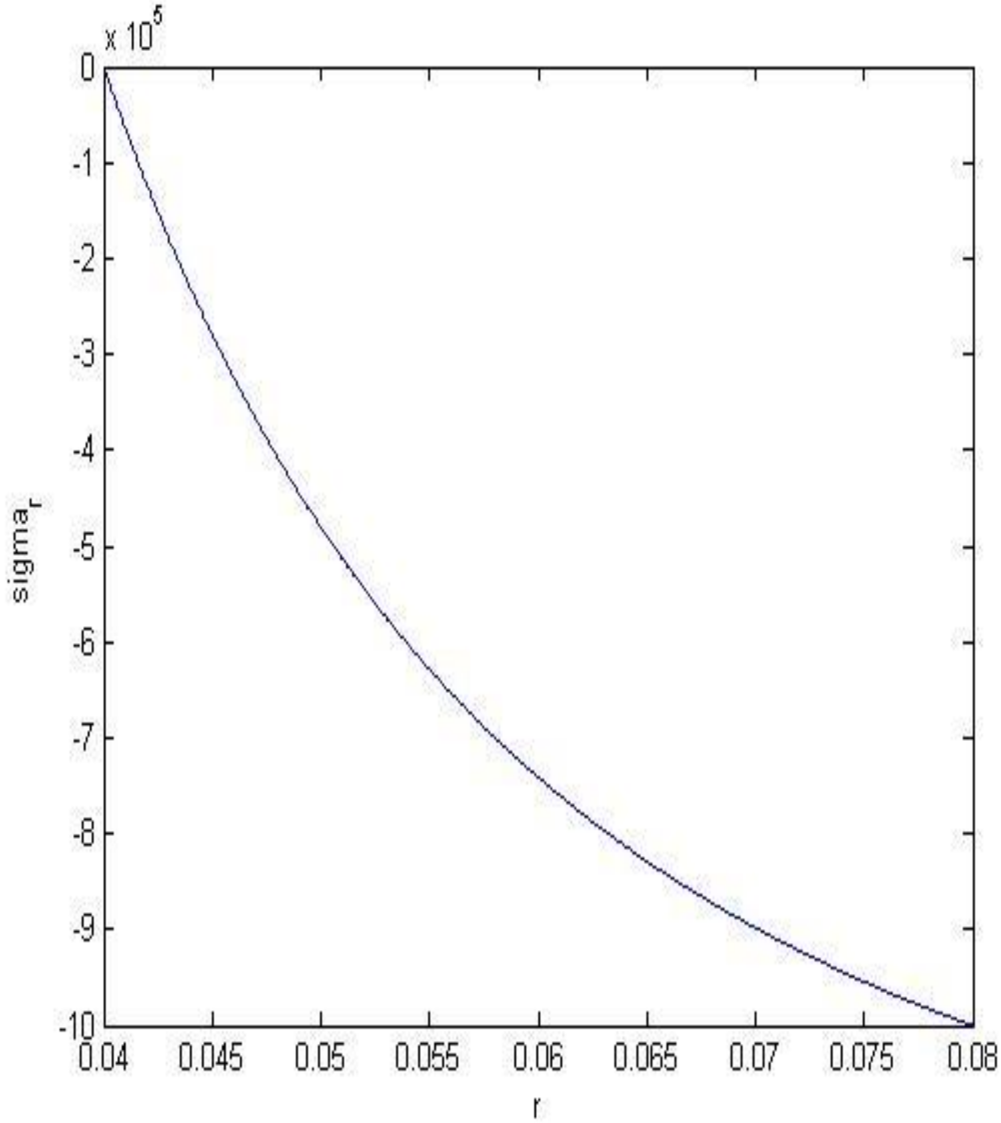


Fig 3.5: Variation of radial stress along radius subjected to external pressure only.

The graph above shows the variation of radial stress along the radius of thick walled cylinder subjected to external pressure. The graph shows that radial stress in this case is a compressive stress as its magnitude is negative throughout the graph.

Program for plotting graph between hoop stress and radius for thick walled cylinder subjected to external pressure only

```
close all
clear all


pa=0;
pb=1000*10^3;
a=0.04;
b=0.08;
r=[a:(b-a)/1000:b];
n=1;
while(n<=1001)
sigma_r(n)= ((pa*a^2 - pb*b^2)/(b^2 - a^2)) - (((a^2*b^2)/r(n)^2)*((pa-pb)/(b^2-a^2)));
sigma_t(n)= ((pa*a^2 - pb*b^2)/(b^2 - a^2)) + (((a^2*b^2)/r(n)^2)*((pa-pb)/(b^2-a^2)));
n=n+1;
end
plot(r,sigma_r)
xlabel('r')
ylabel('sigma_r')
```

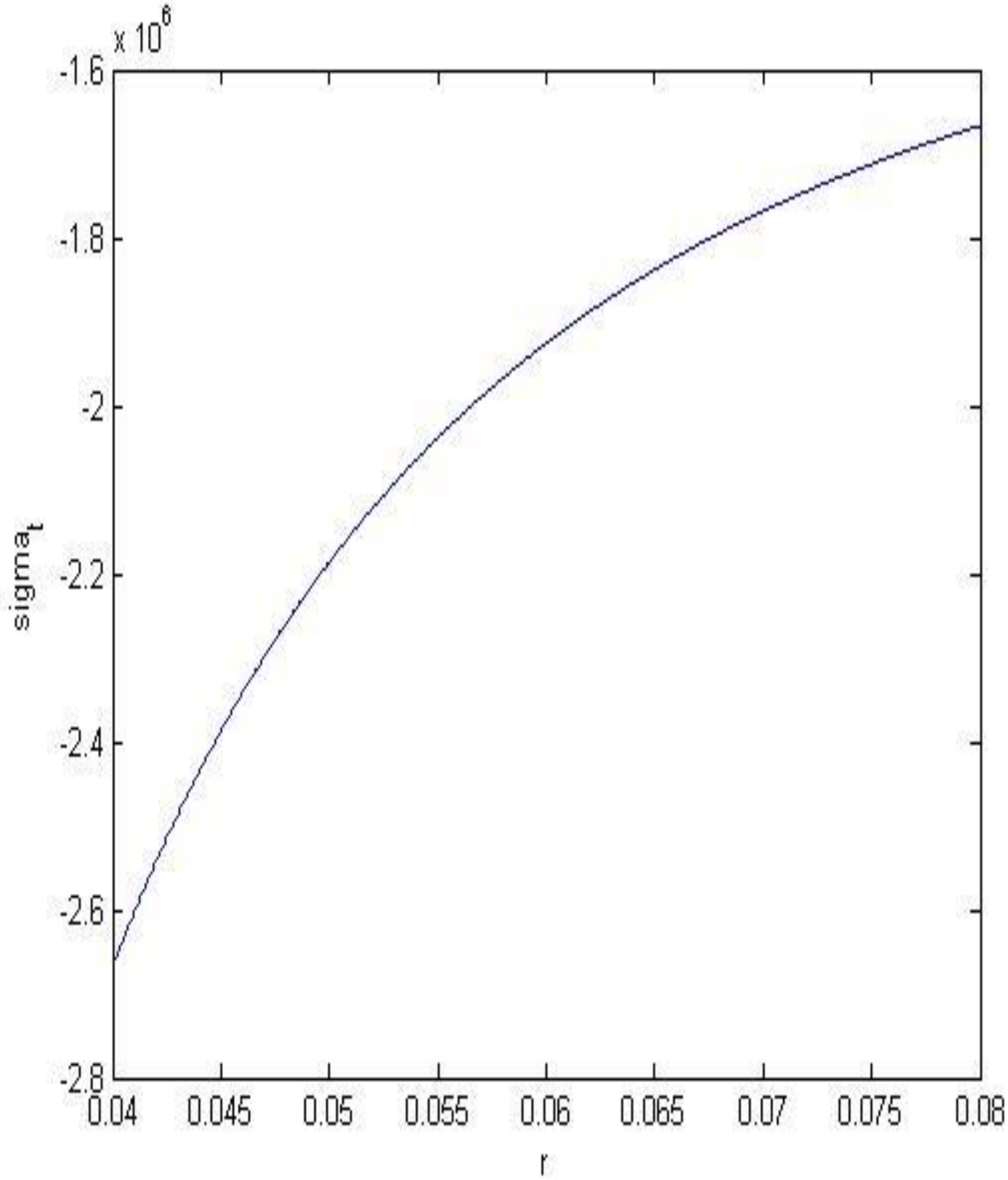


Fig 3.5:Variation of hoop stress along radius subjected to external pressure only.

The graph above shows the variation of hoop stress along the radius of thick walled cylinder subjected to external pressure. The graph shows that hoop stress in this case is a compressive stress as its magnitude is negative throughout the graph.

Program for plotting graph between radial stress and radius for Composite Tubes (Shrink fit)

```
close all
clear all
p=8000*10^3;
a=0.04;
b=0.07;
c=0.12;
r=[a:(c-a)/10000:c];
n=1;
while(r(n)<0.07)
pa=0;
pb=p;
sigma_r(n)= ((pa*a^2 - pb*b^2)/(b^2 - a^2)) - (((a^2*b^2)/r(n)^2)*((pa-pb)/(b^2-a^2)));
 n=n+1;
end
n=n-1;
while(n<=10001)
pa=p;
pb=0;
sigma_r(n)= ((pa*b^2 - pb*c^2)/(c^2 - b^2)) - (((b^2*c^2)/r(n)^2)*((pa-pb)/(c^2-b^2)));
 n=n+1;
end
plot(r,sigma_r,'k')
xlabel('r')
ylabel('sigma_r')
```

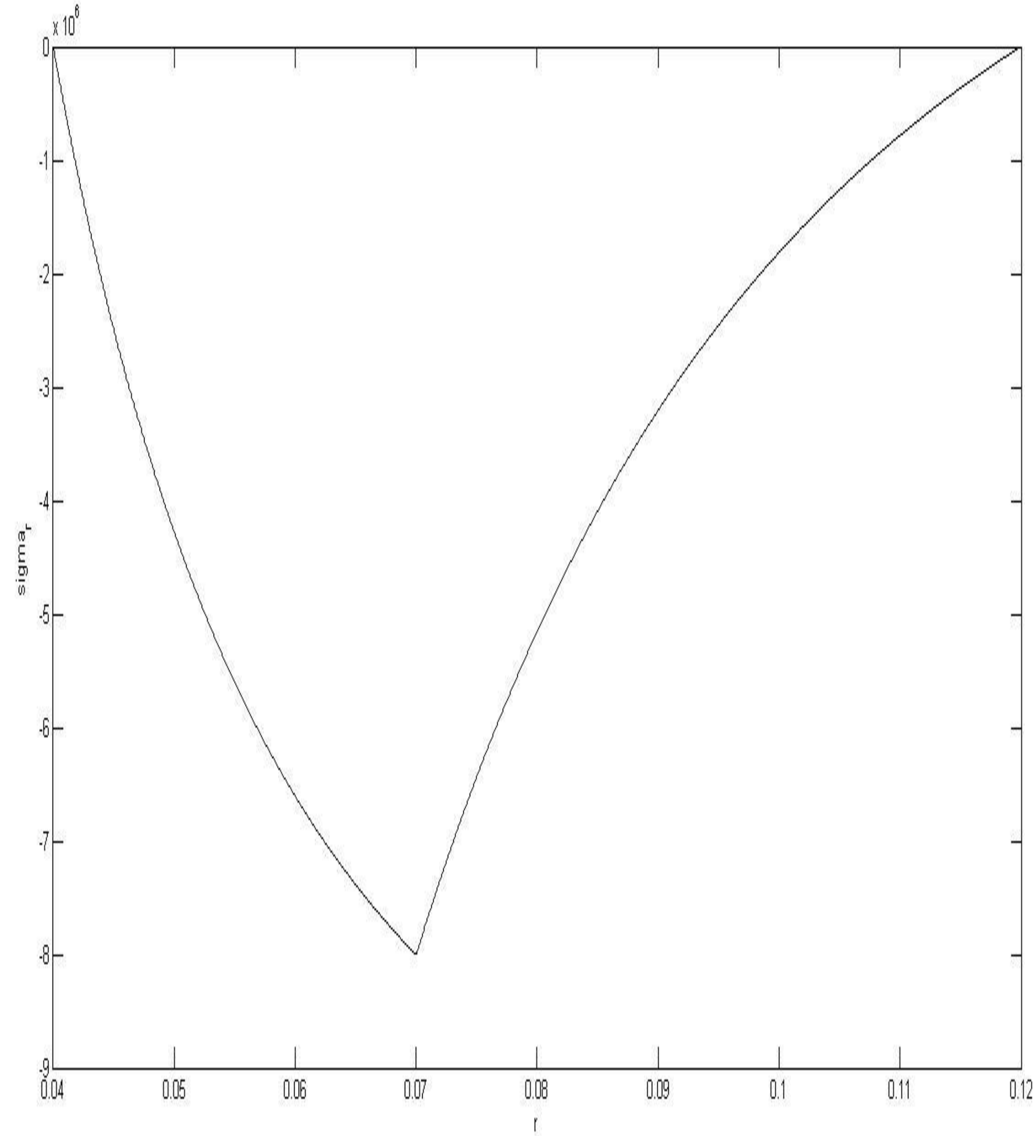


Fig. 3.7: Variation of radial stress along radius in case of shrink fit

The graph above shows the variation of radial stress along the radius in case of shrink fit. The graph shows that radial stress in this case is a compressive stress as its magnitude is negative throughout the graph. It can also be seen that the magnitude of radial stress increases upto the external radius of the inner radius and then decreases till it reaches zero at the external radius of the outer cylinder.

Program for plotting graph between stress and radius for Composite Tubes (Shrink fit)

```
close all
clear all
p=8000*10^3;
a=0.04;
b=0.07;
c=0.12;
r=[a:(c-a)/10000:c];
n=1;
while(r(n)<0.07)
pa=0;
pb=p;
sigma_t(n)= ((pa*a^2 - pb*b^2)/(b^2 - a^2)) + (((a^2*b^2)/r(n)^2)*((pa-pb)/(b^2-a^2)));
n=n+1;
end
n=n-1;
while(n<=10001)
pa=p;
pb=0;
sigma_t(n)= ((pa*b^2 - pb*c^2)/(c^2 - b^2)) + (((b^2*c^2)/r(n)^2)*((pa-pb)/(c^2-b^2)));
 n=n+1;
end
plot(r,sigma_t,'k')
xlabel('r')
ylabel('sigma_t')
```

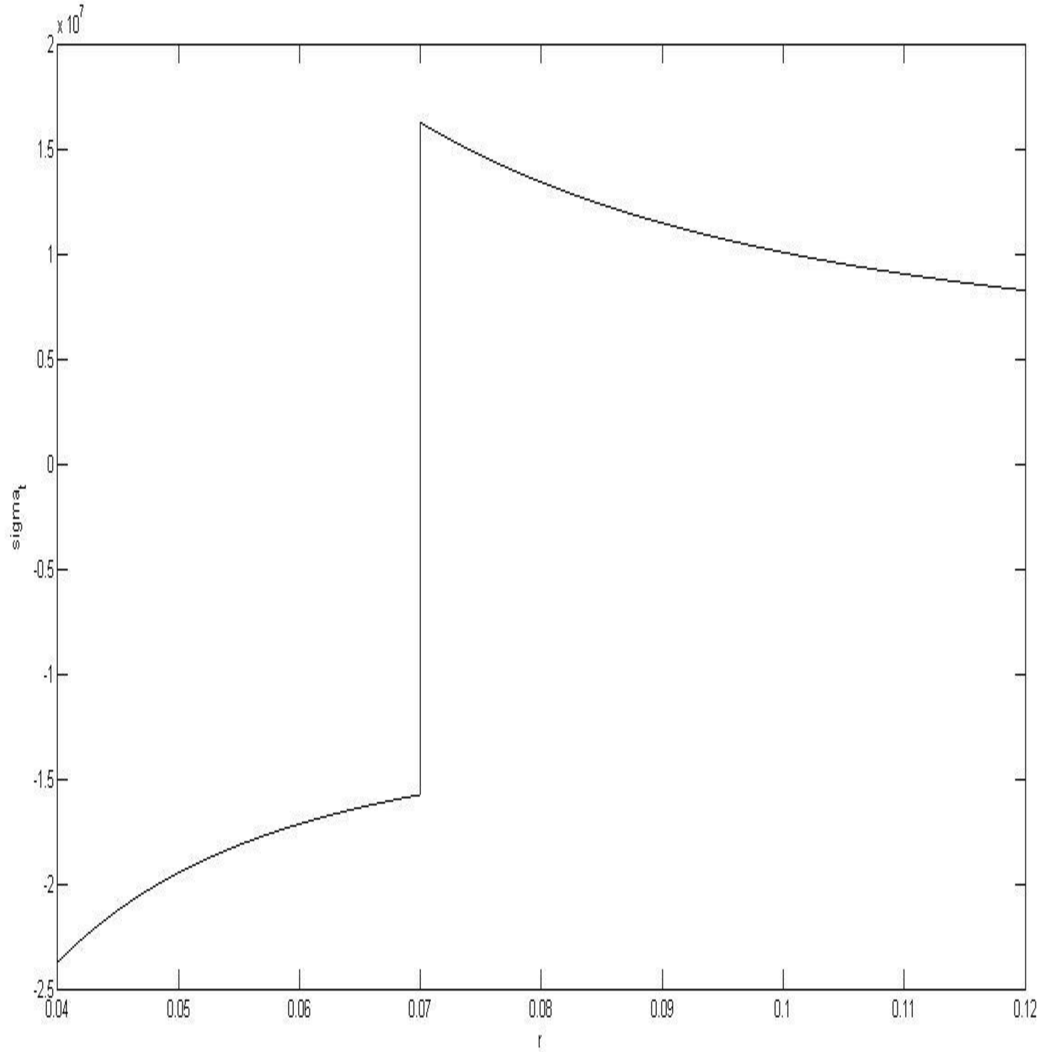


Fig. 3.7: Variation of hoop stress along radius in case of shrink fit

The graph above shows the variation of hoop stress along the radius in case of shrink fit. The graph shows that hoop stress in this case is a compressive stress as its magnitude is negative upto the external radius of the inner cylinder and is a tensile stress from thereon till the external radius of the outer cylinder as its magnitude is positive.

CHAPTER 4

4.1 Summary

An attempt has been made to know the load capacity of a thick walled cylinder . Classical bookwork formulas have been employed to obtain the stress distribution in cylinder subjected to internal and external pressure. Variation of stress across the thickness are shown using Matlab.

4.2 Future Scope of Work

In this Project,Stress analysis is done on simple thick walled cylinder. But in industries there is wide application of thick walled cylinder having holes.Stress analysis of that should be done in future. Plasticity and yield strength of material should be analysed so that we can have better understanding while using. In case of shrink fit there was no external forces ware applied in analysis. In future it should be analysed with exernal forces

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